Journal of Statistical Physics, Vol. 83, Nos. 5/6, 1996

Subtleties in Data Analysis Related to the Size of Critical Region

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Received March 29, 1995; final June 14, 1995

We comment on the analysis of the critical behavior of a layered driven diffusive system recently done by Achahbar and Marro. We discuss why we believe their method of taking the thermodynamic limit and determining the order-parameter exponent β leads to unreliable estimates.

KEY WORDS: Driven diffusive systems; critical behavior; finite-size scaling; computer simulations.

In a recent article, Achahbar and Marro (AM)[1] studied the phase transitions in a model consisting of two stacked layers of a driven lattice gas on square lattices. Each layer is identical to the "standard" driven diffusive system[2] (see ref. 3 for a recent review) with only inplane nearest neighbor interactions, but the layers are coupled via particle hoppings between them. At half-filling, they found two kinds of transitions: a continuous one at a critical temperature T_c, and a discontinuous one at a lower temperature. This comment concerns their analysis of the former transition. First, we will point out that their extrapolation for the order parameter (m) to the thermodynamic limit $(L \to \infty)$ is not justified, especially since this approach contradicts their own finding of different critical exponents for correlations along the horizontal and vertical directions, namely $\nu_h = 0.7 \pm 0.2$ and $\nu_v = 0.4 \pm 0.2$. Second, we believe that, in their determination of the order-parameter exponent β, AM did not address the issue of the critical region. Finally, their Fig. 17 misrepresents the data of ref. 4 and is quite misleading.

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0022-4715/96/0600-1219$09.50/0

For an isotropic system of size $L_h \times L_v$ near a critical point, scaling functions such as that of m generally depend on the aspect ratio L_h/L_v. It is well known (see e.g., ref. 5) that the behavior for a finite ratio is quite different from that for, say, $L_h/L_v \to 0$. On the other hand, scaling functions for an anisotropic system depend on a further scaling variable, $S \equiv L_h^{\nu_v/\nu_h}/L_v$, especially if the correlation lengths along different directions ξ_h and ξ_v, diverge at different rates[4]. Exact results on the Kasteleyn model show precisely this behavior[6]. Clearly, what the "thermodynamic limit" means will then depend on how it is taken with regard to S. Parallel to keeping the aspect ratio fixed for isotropic systems, the least complicated way to approach this limit is to keep S fixed. In particular, if square shapes of various sizes are used, then $S \to 0$ and extra singularities can arise[4] and m

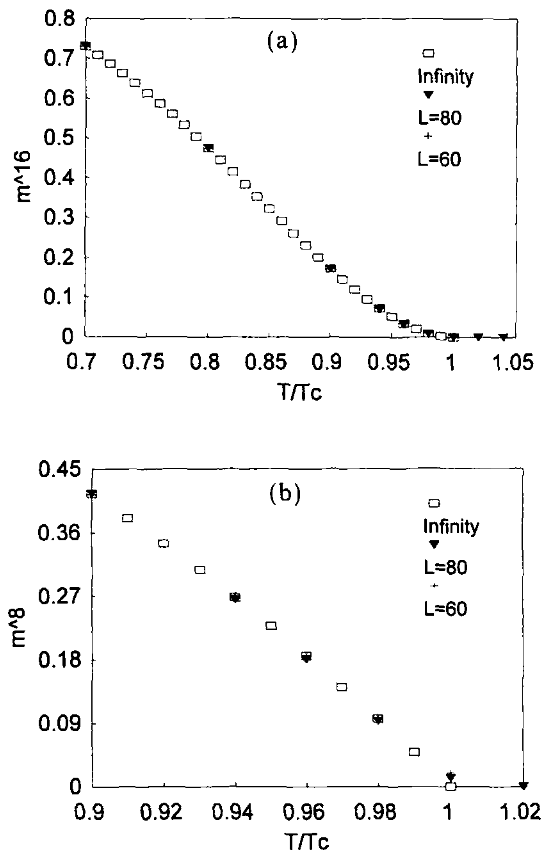

Fig. 1. Plot of m^p vs. T/T_c for the 2D Ising model: (a) $p = 16$, assuming a wide critical region; (b) $p = 8$, assuming a narrow critical region suggested by the fluctuation in m.

is not likely to scale simply as $1/L$ (at fixed temperature), as assumed by AM. Since the effect of $S \to 0$ has not been taken into account, the meaning of their scaling plot is considerably clouded.

To determine β, AM identified $1/\beta$ as the value p which yields the best linear plot of m^p versus T. We contend that this procedure is unreliable, despite its ubiquity, because the estimate depends crucially on the size of the critical region (CR) assumed. To demonstrate our point, we apply this method to the two-dimensional (2D) Ising model, for which $m(T; L_v = L_h = \infty)$ and $\beta = 1/8$ are known exactly. Assuming a wide CR as in Fig. 1a (with the same 30% range as in AM's Fig. 6), one would arrive at $\beta \approx 1/16$, along with a low estimate of T_c. The correct values are recovered only if a smaller CR is used, as in Fig. 1b.

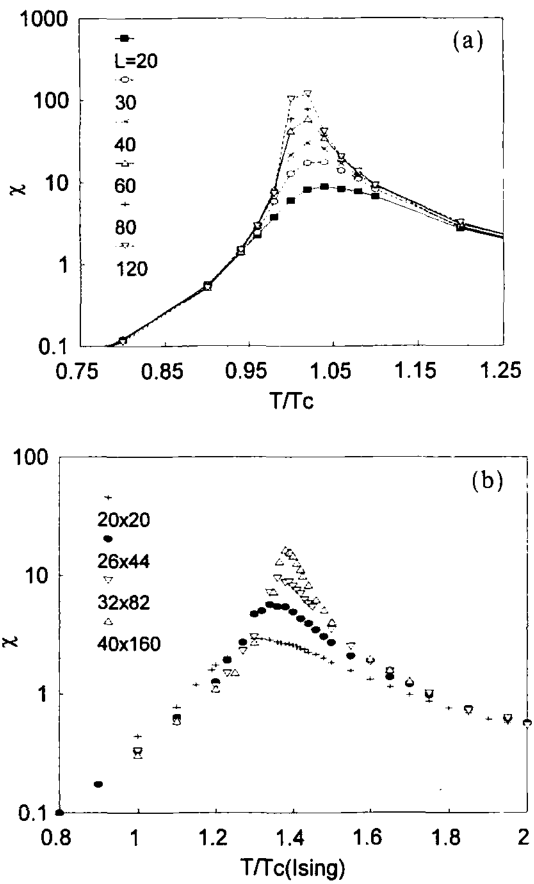

Fig. 2. Susceptibility $\chi \equiv (L_v L_h / T)(\langle m^2 \rangle - \langle m \rangle^2)$ for (a) the 2D Ising model and (b) the one-layer driven diffusive system, showing the interval over which m should scale.

Although the width of the CR is generally unknown and it probably depends on the thermodynamic average in question, we may estimate it from the corresponding fluctuations. For example, simulation data show that the fluctuation of m in the Ising model becomes independent of L outside the range $0.9 \leqslant T/T_c \leqslant 1.2$ (see Fig. 2a). For a typical L, the fluctuations outside this region are at least ten times smaller than the peak value. Thus, one can safely conclude that the low-T data in Fig. 1a are outside the scaling regime, so that the estimates of both β and T_c are unreliable. Turning to the fluctuations in the driven models, we find that they are very small at the lower end of the temperature scale in Fig. 6 of AM (cf. Fig. 2b). Therefore, we believe that their β is at best an effective exponent characterizing the low-T behavior.

In their analysis of the one-layer system (in Section 3), they claimed to find "clear evidence depicted in Fig. 17" of the departure from scaling for the field-theoretic predictions. Close examination, however, reveals an alarming approach to arrive at this conclusion, namely, exploiting data from *extremely low* T. In particular, for systems of 26×44 and 20×20, temperatures as low as 0.6 and 0.1, respectively, were used! At those temperatures, m saturates to unity, correlation lengths are of $O(1)$, so that fluctuations are negligible and become independent of system sizes. Thus, deviations from scaling for such data are totally expected. Presenting such data along side those for temperatures closer to T_c hardly tarnishes the good scaling behavior of the latter. Fig. 3 should make our point clear: applying AM's procedure and reasoning to the 2D Ising model, one would

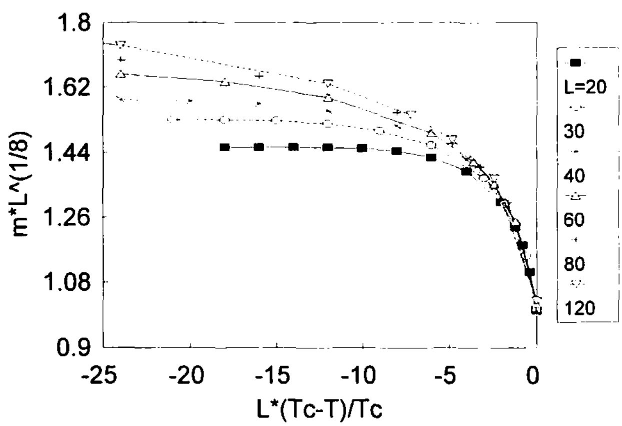

Fig. 3. Finite-size scaling plot $mL^{\beta/\nu}$ vs. $L^{1/\nu}(T_c - T)/T_c$ for 2D Ising model; the exact values $\beta = 1/8$, $\nu = 1$, and T_c are used. The lowest T/T_c values are, analogous to AM's Fig. 17; 0.1 ($L = 20$), 0.3 ($L = 30$), 0.4 ($L = 40$), 0.6 ($L = 60$), 0.7 ($L = 80$), and 0.8 ($L = 120$).

have to exclude $\beta = 1/8$! On the other hand, following our principle for the driven case, we see that the fluctuations of m become largely independent of L outside $1.2 \leqslant T \leqslant 1.8$ (see Fig. 2b), and, using data *within* this interval, we indeed observe scaling behavior with the parameters of ref. 4.

In response to their assertion that "the size more than the shape of the system matters" in Section 4, we call the reader's attention to the recent work by Wang[7], who carried out extensive simulations and investigated systematically the effect of S. Based on much larger lattice sizes and much longer runs than any previous study, his results support the conclusion of ref. 4.

Finally, AM's argument in favor of a "universality class" for several nonequilibrium systems in ref. 1 and elsewhere[8] is not well founded, even if their conclusion may turn out to be correct. Measuring two exponents (β, ν), using the same unreliable methods, hardly justifies the assignment of different systems in one class.

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Communicated by J. L. Lebowitz